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Travelling Waves Solution of the Unsteady Flow Problem of a Collisional Plasma Bounded by a Moving Plate

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Abstract: The extension of the previous paper [Can. J. Phys. Vol. 88, (2010), 501–511] has been made. Therefore, the effect of the neutral atoms collisions with electrons and with positive ions is taken into consideration, which was ignored, for the sake of simplicity, in the earlier work. Thus, we will have multi-collision terms (electron–electron, electron–ion, electron– neutral) instead of one term, as was studied before for the sake of facilitation. These collision terms are needed to obtain the real physical situation. The new procedures will increase the ability of the research applications. This study is based on the solution of the BGK (Bhatnager–Gross–Krook) model of the nonlinear partial differential Boltzmann equations coupled with Maxwell's partial differential equations. The initial-boundary value problem of the Rayleigh flow problem applied to the system of the plasma (positive ions + electrons+ neutral atoms), bounded by a moving plate, is solved. For this purpose, the traveling wave solution method is used to get the exact solution of the nonlinear partial differential equations system. The ratios between the different contributions of the internal energy changes are predicted via the extended Gibbs equation for both dia-magnetic and para-magnetic plasma. The results are applied to a typical model of laboratory argon plasma. 3D-Graphics illustrating the calculated variables are drawn to predict their behavior and the results are discussed.

Keywords: Rayleigh Flow Problem, Charged Gas, Boltzmann Equation, Maxwell Equations, Exact Solution, Boltzmann H-Theorem, Internal Energy, Extended Gibbs Formula

1. Introduction

Partially ionized plasmas at low gas pressure have found a wide range of applications in many branches of contemporary technology. Gas discharge lighting, manufacturing of semiconductor chips, plasma treatment of materials [1, 2], Astrophysics [3], nanotechnology, plasma chemistry, and bio-medical treatments are the most known applications of low temperature plasma [4]. Plasma (charged gases) based technology underpins some of the world's largest industries producing a significant proportion of the world's global commercial products including; computers, cell phones, automobiles, aero planes, paper and textiles [5]. Foremost among these is the electronics industry, in which plasma-based processes are indispensable for the manufacture of ultra-large-scale integrated microelectronic circuits. The Boltzmann equation represents a well-defined model to describe the motion of plasma, when microscopic effects must be considered. This is the case of an electron flow in MEMS (Micro-Electro-Mechanical Systems).

Abourabiia and Abdel Wahid [6], in the framework of irreversible thermodynamics, examined the characteristics of the Rayleigh flow problem of a rarified electron gas extracted from neutral atoms and proved that, it obeys the entropic behavior for gas system but with an approximate solution and inaccurate formula of the collision frequency. In the present paper, the enhancement and improvement of this study is done. The present study has obvious peculiarities in comparison with the previous one [6]. In principle, the complete and accurate formulas of collision frequencies are used, avoid the discontinuity in the solution using Laplace transformation, used in [6], and introduce the complete value

of variables without any cutoff caused from the small parameters method as in [6] and in this study we have no restriction on the non-dimensional parameter like [6].

The kinetic theory has contributed not only to the understanding of nonequilibrium transport phenomena in gases, but also to the development of general nonequilibrium statistical physics. It is well accepted that the Boltzmann equation [7-20] is one of the most reliable kinetic models for describing nonequilibrium phenomena in gas phase. Following its success and usefulness, The Boltzmann equation is widely used in order to describe various gas-phase transport phenomena such as plasma gases, granular gases, polyatomic gases, relativistic gases and chemically reacting gases [11-20]. The kinetic equation of gas flow based on the Boltzmann equation, has obvious peculiarities in comparison with the macroscopic description found by using the Navier–Stokes equations, see [15]. Since it is very difficult to solve the full Maxwell-Boltzmann equations, various approximations have been suggested, such as the Chapman-Enskog procedure, Krook's model, and Lee's moment method [13-20] for the solution of the Boltzmann's equation.

The objective of this paper is to solve the initial-boundary value problem of the Rayleigh flow problem applied to the system of the plasma to determine the macroscopic parameters such as the mean velocity, shear stress, viscosity coefficient together with the induced electric and magnetic fields. Using the estimated distribution functions, it is of fundamental physical importance to study the irreversible thermodynamic behavior of the diamagnetic and paramagnetic plasma, so that the predictions of the entropic behavior and related thermodynamic functions are investigated. The results are applied to a typical model of laboratory Argon plasma. The results agree with proceeding theoretical studies are illustrated.

2. The Physical Problem and Mathematical Formulation

Let us assume that the upper half of the space ($y \geq 0$), which is bounded by an infinite flat plate (y = 0), is filled with a multi-component plasma. The plasma is initially in absolute equilibrium and the wall is at rest. Then the plate starts to move suddenly in its own plane with a velocity ($U_0 e^{-\alpha t}$) along the x-axis (U_0 and α are constants). Moreover, the plate is considered impermeable, uncharged, and an insulator. The whole system (electrons + ions + neutrals +plate) is kept at a constant temperature. All physical quantities are defined in the nomenclature.

Let the forces $\vec{f}_e$ acting on each electron; be given by [22-24]:

$$\vec{f}_e = -e\vec{E}_e - \frac{e}{c_o}(\vec{c} \wedge \vec{B}_e) \qquad (1)$$

By taking

$$\vec{V} \equiv (V_x, 0, 0), \vec{J} \equiv (qnV_x, 0, 0), \vec{E} \equiv (E_x, 0, 0) \text{ and } \vec{B} \equiv (0, 0, B_z) , \quad (2)$$

We assume that V_x, E_x, B_z, and J_x are functions of y and t. This choice satisfies Maxwell's equations. The distribution function $F (y, c, t)$ of the particles for the plasma gas can be obtained from the kinetic Boltzmann's equation, which can be written in the BGK (Bhatnager–Gross–Krook) model as:

$$\frac{\partial F_e}{\partial t} + \vec{c} \cdot \frac{\partial F_e}{\partial \vec{r}} + \frac{\vec{f}_e}{m_e} \cdot \frac{\partial F_e}{\partial \vec{c}} = \nu_{ee}(F_{0e} - F_e) + \nu_{ei}(F_{0i} - F_e) + \nu_{en}(F_{0n} - F_e) \qquad (3)$$

for electrons,

Where $F_{0\alpha} = n_\alpha (2\pi R T_\alpha)^{-\frac{3}{2}} \exp\left(\frac{-(\vec{c} - \vec{V}_\alpha)^2}{2RT_\alpha} \right)$

The quantities $n_\alpha, \vec{V}_\alpha$ and T_α are the number density, mean drift velocity, and effective temperature obtained by taking moments of F_α.

The particles are reflected from the plate with a full velocity accommodation i. e., the plasma particles are reflected with the plate velocity so that the boundary conditions are:

$V_{x2}(0, t) = U_0 e^{-\beta t} \text{ for } t > 0$, where $V_{x2} = V_x \text{ as } c_y > 0$

and V_x is finite as $y \rightarrow \infty$ for electrons [5].

Substituting from Eqs. (1, 2) into Eq. (3) one obtains;

$$\frac{\partial F_e}{\partial t} + c_y \frac{\partial F_e}{\partial y} - \frac{eB_{ez}}{m_e c_0}(c_y \frac{\partial F_e}{\partial c_x} - c_x \frac{\partial F_e}{\partial c_y}) + \frac{eE_{ex}}{m_e} \frac{\partial F_e}{\partial c_x} = \nu_{ee}(F_{0e} - F_e) + \nu_{ei}(F_{0i} - F_e) + \nu_{en}(F_{0n} - F_e) , \qquad (4)$$

for electrons, where ν_{ee}, ν_{ei} and ν_{en} are electron-electron, electron-ion, and neutral-electron collision frequencies respectively, which are given by [23-24, 31]:

$$\nu_{ee} = \left(\frac{4\sqrt{\pi} n_e e^4 Log[\Lambda_{ee}]}{3\sqrt{m_e} K_B^{\frac{3}{2}} T_e^{\frac{3}{2}}} \right), \nu_{ei} = \left(\frac{4\sqrt{2\pi} n_i e^4 Z^2 Log[\Lambda_{ei}]}{3\sqrt{m_e} K_B^{\frac{3}{2}} T_e^{\frac{3}{2}}} \right) \text{ and}$$

$$\nu_{en} = \left(\frac{4\sqrt{\pi m_n} n_e e^4 Log[\Lambda_{ee}]}{3m_e K_B^{\frac{3}{2}} T_e^{\frac{3}{2}}} \right) \qquad (5)$$

where $\lambda_{De} = \lambda_{Di} = \lambda_D$, $Log[\Lambda] = Log[4\pi n \lambda^3{}_D]$ and Z are the Coulomb Logarithms and the degree of ionization respectively.

The model of the cone of influence suggested by Lee's moment method [25] for the solution of the Boltzmann's equation is employed here. Let us write the solution of Eq. (4), as suggested by Kashmarov [7] in the form:

$$F = \begin{cases} F_1 = n(2\pi RT)^{-\frac{3}{2}}\left(1 + \dfrac{c_x V_{x1}}{RT}\right)\exp\left(\dfrac{-c^2}{2RT}\right) & \text{for } c_y < 0 \\[3mm] F_2 = n(2\pi RT)^{-\frac{3}{2}}\left(1 + \dfrac{c_x V_{x2}}{RT}\right)\exp\left(\dfrac{-c^2}{2RT}\right) & \text{for } c_y > 0 \end{cases} , \quad (6)$$

where V_{x1} and V_{x2} are two unknown functions of time t and the single distance variable y.

Using Grad's moment method [26] multiplying Eq. (4) by $Q_j(\vec{c})$ and integrating over all values of $\vec{c}$, we obtain the transfer equations for electrons in the form:

$$\frac{\partial}{\partial t}\int Q_j F_e d\underline{c} + \frac{\partial}{\partial y}\int c_y Q_j F_e d\underline{c} + \frac{eE_{xe}}{m_e}\int F_e \frac{\partial Q_j}{\partial c_x}d\underline{c} + -\frac{eB_{ze}}{m_e c_0}\int (c_x \frac{\partial Q_j}{\partial c_y} - c_y \frac{\partial Q_j}{\partial c_x})d\underline{c}$$

$$= v_{ee}\int (F_{0e} - F_e)Q_j d\underline{c} + v_{ei}\int (F_{0i} - F_e)Q_j d\underline{c} + v_{en}\int (F_{0n} - F_e)Q_j d\underline{c} \tag{7}$$

The integrals over the velocity distance are evaluated from the relation [8-10],

$$\int Q_j(\vec{c})Fd\underline{c} = \int_{-\infty}^{\infty}\int_{-\infty}^{0}\int_{-\infty}^{\infty} Q_j F_1 d\underline{c} + \int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{-\infty}^{\infty} Q_j F_2 d\underline{c} \tag{8}$$

where $Q_j = Q_j(\vec{c}), j = 1,2$ and $d\underline{c} = dc_x dc_y dc_z$, where c_x, c_y and c_z are the particles velocities components along x, y and z- axes, respectively. Moreover, E and B may be obtained from Maxwell's equation, for electrons

$$\frac{\partial E_{xe}}{\partial y} - \frac{1}{c_0}\frac{\partial B_{ze}}{\partial t} = 0 , \tag{9}$$

$$\frac{\partial B_{ze}}{\partial y} - \frac{1}{c_0}\frac{\partial E_{xe}}{\partial t} - \frac{4\pi e n_e}{c_0}V_{xe} = 0 . \tag{10}$$

where $n = \int Fd\underline{c}$, $nV_x = \int c_x Fd\underline{c}$, with the initial and boundary conditions:

$$\left.\begin{array}{l} E_x(y,0) = B_z(y,0) = 0 \quad , \\ E_x(y,t) \text{ and } B_z(y,t) \text{ are finite as } y \to \infty. \end{array}\right\} \tag{11}$$

We introduce the dimensionless variables defined by:

$$\left.\begin{array}{l} t = t'\tau_{ee} \ , y = y'\left(\dfrac{\tau_{ee}V_{Te}}{\sqrt{2\pi}}\right), V_x = V_x' V_{Te} \ , \tau_{xy} = \tau_{xy}' V_{Te} \ , M_a = \dfrac{U_0}{V_{Te}} \\[4mm] B_z = B_z'\left(\dfrac{\sqrt{2\pi}m_e c_0}{e\tau_{ee}}\right), E_x' = E_x'\left(\dfrac{m_e V_{Te}}{e\tau_{ee}}\right) \ , \rho = nm \ , V_{Te} = \sqrt{\dfrac{2K_B T_e}{m_e}} \\[4mm] \varepsilon = \dfrac{m_e}{m_i}, dU = dU'\left(K_B T_e\right) \text{ and } F_j = F_j' n_e (2\pi RT_e)^{-\frac{3}{2}}, j = 0,1,2 \end{array}\right\} \tag{12}$$

For $M_a^2 \ll 1$ (low Mach number), we can assume that the density and the temperature variation at each point of the flow and at any time are negligible, i. e., $n_\alpha = 1 + O(M_a^2)$ and. $T_\alpha = 1 + O(M_a^2)$ Let

$$V_x = \frac{1}{2}(V_{x1} + V_{x2}) \ , \ \tau_{xy} = \frac{P_{xy}}{\rho U_0 \sqrt{RT_e/2\pi}} = (V_{x2} - V_{x1}). \tag{13}$$

Where P_{xy} is the shear stress [5, 7] defined by

$$P_{xy} = m\int (c_x - V_x)c_y Fd\underline{c}$$

Using the dimensionless variable, Eq. (7) for $Q_1 = c_x$ and $Q_2 = c_x c_y$ become:

$$\frac{\partial V_{xe}'}{\partial t'} + \frac{\partial \tau_{xye}'}{\partial y'} - E_{xe}' = 0 , \tag{14}$$

$$\frac{\partial \tau_{xye}'}{\partial t'} + 2\pi \frac{\partial V_{xe}'}{\partial y'} + \tau_{xye}' = 0 . \tag{15}$$

with the initial and boundary conditions:

$$\left.\begin{array}{l} V_x'(y',0) = \tau_{xy}'(y',0) = 0, \\ 2V_x'(0,t') + \tau_{xy}'(0,t') = 2M_p e^{-\beta_1 t'} \\ V_x' \text{ and } \tau_{xy}' \text{ are finite as } y \to \infty, \beta_1 = \beta\tau_{ee}. \end{array}\right\} \tag{16}$$

For the sake of simplicity, henceforth, we drop the dash over the dimensionless variables. Therefore, we have the

following initial-boundary value problem for electrons (neglecting the displacement current) [22]:

$$\frac{\partial V_{xe}}{\partial t} + \frac{\partial \tau_{xye}}{\partial y} - E_{xe} = 0, \tag{17}$$

$$\frac{\partial \tau_{xye}}{\partial t} + 2\pi \frac{\partial V_{xe}}{\partial y} + \left(\frac{v_{ee}}{v_{ee}} + \frac{v_{ei}}{v_{ee}} + \frac{v_{en}}{v_{ee}}\right)\tau_{xye} = 0, \tag{18}$$

$$\frac{\partial E_{xe}}{\partial y} - \frac{\partial B_{ze}}{\partial t} = 0, \tag{19}$$

$$\frac{\partial B_{ze}}{\partial y} - \alpha_{0e} V_{xe} = 0, \text{ where } \alpha_{0e} = \left(\frac{V_{Te}^2 \tau_{ee}^2 e^2 n_e}{m_e c_0^2}\right). \tag{20}$$

With the initial and boundary conditions

$$\left.\begin{array}{l} V_{xe}(y,0) = \tau_{xye}(y,0) = E_{xe}(y,0) = B_{ze}(y,0) = 0, \\ 2V_{xe}(0,t) + \tau_{xye}(0,t) = 2Ma\,e^{-\beta_1 t}, \text{ for } t > 0 \; ; \\ V_{xe}, \tau_{xye}, E_{xe} \text{ and } B_{ze} \text{ are finite as y} \to \infty. \end{array}\right\} \tag{21}$$

We can reduce the basic Eqs. (17) - (20), after simple algebraic manipulations to a single equation:

$$\frac{\partial^4 V_{xe}(y,t)}{\partial t^2 \partial y^2} - 2\pi \frac{\partial^4 V_{xe}(y,t)}{\partial t^4} - A_c \frac{\partial^3 V_{xe}(y,t)}{\partial t \partial y^2} - \alpha_0 \frac{\partial^2 V_{xe}(y,t)}{\partial t^2} - \alpha_0 A_c \frac{\partial V_{xe}(y,t)}{\partial t} = 0 \tag{22}$$

where $A_c = \left(1 + \frac{v_{ei}}{v_{ee}} + \frac{v_{en}}{v_{ee}}\right)$.

Solution of the Initial-Boundary Value Problem
We will use the traveling wave solution method [28-29] considering

$$\xi = ly - mt. \tag{23}$$

Such that to make all the dependent variables as functions of ξ. Here *l* and *m* are transformation constants, which do not depend on the properties of the fluid but as parameters to be determined by the boundary and initial conditions [28-29]. From Eq. (23) we get the derivatives:

$$\frac{\partial}{\partial t} = -m \frac{\partial}{\partial \xi}, \frac{\partial}{\partial y} = l \frac{\partial}{\partial \xi} \text{ and } \frac{\partial^a}{\partial t^a} = (-1)^a m^a \frac{\partial^a}{\partial \xi^a}, \frac{\partial^a}{\partial y^a} = l^a \frac{\partial^a}{\partial \xi^a}, \tag{24}$$

where *a* is a positive integer.
Substituting from Eqs. (23-24) into Eqs. (22) to get:

$$\left(m^2 l^2 - 2\pi m^4\right)\frac{d^4 V_{xe}(\xi)}{d\xi^4} + ml^2 A_c \frac{d^3 V_{xe}(\xi)}{d\xi^3} - \alpha_0 m^2 \frac{d^2 V_{xe}(\xi)}{d\xi^2} + \alpha_0 m A_c \frac{dV_{xe}(\xi)}{d\xi} = 0 \tag{25}$$

The boundary and initial conditions become:

$$\left.\begin{array}{l} E_{xe}(\xi = 0) = B_{ze}(\xi = 0) = \tau_{xye}(\xi = 0) = 0, \\ 2V_{xe}(\xi = -m) + \tau_{xye}(\xi = -m) = 2M_p e^{-\beta_1} \text{ at } y = 0 \text{ ,e.g., } t = 1 \\ V_{xe}, \tau_{xye}, E_{xe} \text{ and } B_{ze} \text{ are finite as } \xi \to -\infty. \end{array}\right\} \tag{26}$$

Now we have an ordinary differential Eq. (25) with the boundary and initial conditions (26).

Exactly the help of symbolic computer software, with their boundary and initial conditions (26), can solve the ordinary fourth order homogeneous differential Eq. (25). The sought solutions will be applied to a typical model of laboratory Argon plasma.

3. The Non-Equilibrium Thermodynamic Predictions of the Problem

The application of Onsager's principle to a thermodynamic system as a whole or to an individual part of it (locally) is based on a representation of the entropy production as a finite sum of products of fluxes and forces. In addition, on linearity of the description of the nonequilibrium state of the system or its parts, this gives a linear connection between the fluxes and the forces. The symmetry of the coefficients of this connection constitutes Onsager's principle [11-15]. Local application of the principle to a fluid particle of the plasma gas requires its state to be near local equilibrium and its description by a finite set of macroscopic parameters. The problems of the thermodynamics of irreversible processes continue to present great importance. This is associated both with the general theoretical importance of this theory and its numerous applications in various branches of science. Starting from the essentials of the H-theorem, we begin with the evaluation of the entropy per unit mass S. It is written in dimensionless form as [11-15]:

$$S = -\int F_e \ln F_e \, d\underline{c} = -\left(\int F_{1e} \ln F_{1e} \, d\underline{c} + \int F_{2e} \ln F_{2e} \, d\underline{c}\right) = -\pi^{\frac{3}{2}}\left[(V_{x1}^2 + V_{x2}^2) - \frac{3}{2}\right]. \tag{27}$$

In addition, we get the entropy flux component in the y-direction:

$$J_y^{(S)} = -\int c_y F_e \ln F_e \, d\underline{c} = -\left(\int c_y F_{1e} \ln F_{1e} \, d\underline{c} + \int c_y F_{2e} \ln F_{2e} \, d\underline{c}\right) = \left[\pi(V_{x1}^2 + V_{x2}^2)\right] \tag{28}$$

The law of entropy production [8, 28] is written in the local form as:

$$\sigma = \frac{\partial S}{\partial t} + \vec{\nabla} \bullet \overline{J^{(S)}} . \tag{29}$$

Following the general theory of irreversible thermodynamics we could estimate the thermodynamic force [6, 8, 10] corresponding to the plate Mach number, presented as a time dependent controlling parameter

$$X_{e11} = \frac{U_w}{\sqrt{2RT_0}} = \frac{U_0 \exp[-\beta_1 t]}{\sqrt{2RT_0}} = Ma \exp[-\beta_1 t]. \tag{30}$$

On the other hand, the relationship between the entropy production, thermodynamic fluxes and forces has the form [8, 10]:

$$\sigma = \sum_l J_l X_l . \tag{31}$$

Near the thermodynamic equilibrium, the following linear relation between the fluxes and the forces holds:

$$J_l = \sum_j L_{lj} X_j ,$$

where L_{lj} represent the so-called phenomenological coefficients, which have to fulfill the Onsager reciprocal relation, together with the condition of the Onsager's [10]:

$$L_{lj} = L_{jl}, \quad L_{ll} \geq 0 .$$

In this study, this will be satisfied by the coefficient

$$L_{11} \geq 0 . \tag{32}$$

To study the internal energy change for the system we introduce the extended Gibbs relation [32]. It includes the electromagnetic field energy as a part of the whole energy balance, which distinguish the charged gas into paramagnetic and diamagnetic ones. According to whether if there are unpaired electrons in the molecular orbital diagram, the gas is paramagnetic, or if all electrons are paired, the gas is considered as a diamagnetic one. To get the work term in the first law of thermodynamics, we should write the internal energy balance including the electromagnetic field energy as follows:

A) For paramagnetic plasma; the internal energy change is expressed in terms of the extensive quantities S, P and M, which are the thermodynamic coordinates corresponding to the conjugate intensive quantities T, E and B respectively. The three contributions in the internal energy change in the Gibbs formula:

$$dU = dU_S + dU_{pol} + dU_{paa}, \tag{33}$$

where

$dU_S = TdS$ is the internal energy change due to variation of the entropy,

$dU_{pol} = E \, dP$ is the internal energy change due to variation of polarization, and

$dU_{para} = B \, dM$ is the internal energy change due to the variation of magnetization, here m is calculated from the equation [3, 29]

$$\frac{\partial S}{\partial m} = -\frac{B}{T} \Rightarrow M = -\int \left(\frac{T}{B}\frac{\partial S}{\partial y}\right)_t dy . \tag{34}$$

Introducing the dimensionless variables $U' = \frac{U}{KT}, M' = M\left(\frac{1}{e\,\tau_{ei}V_{Te}}\right), p' = p\left(\frac{1}{e\tau_{ee}V_{Te}}\right)$ in the Gibbs formula to get (after dropping the primes)

$$dU = dS + f_1 E \, dp + f_1 B \, dM . \tag{35}$$

B) On the other hand, if the plasma is diamagnetic; the internal energy change due to the extensive variables S, P and B represent the thermodynamic coordinates conjugate to the intensive quantities T, E and M respectively, therefore we have three contributions in the internal energy change in the Gibbs formula given by:

$$dU = dU_S + dU_{pol} + dU_{dia}, \tag{36}$$

where $dU_{dia} = -M \, dB$ is the internal energy change due to the variation of the induced magnetic induction, where $M = T\frac{\partial S}{\partial B}$ [6, 32].

Hence, the dimensionless form for dU in this case takes the form:

$$dU = dS + f_1 E dp - f_1 M \, dB , \tag{37}$$

where

$$f_1 = \left(\frac{m_e V_{Te}^2}{KT_0} \right),\, dS = \left(\frac{\partial S}{\partial r} \right)\, \delta y + \left(\frac{\partial S}{\partial t} \right)\, \delta t \; ; \; \delta y = 0.01 , \;\; \delta t = 0.01 .$$

4. Results and Discussions

In this problem, the unsteady behavior of a rarefied electron gas is studied based on the kinetic theory of irreversible processes using the exact traveling wave analytical solution method via the BGK model of the Boltzmann equation with the exact value of electro-electron collision frequency. Those computations are performed according to typical data for electron gas in Argon plasma [32, 33, 34] as a paramagnetic medium in the case where the argon gas loses single electrons or as a diamagnetic medium in the case where the argon gas loses electron pairs, depending on the ionizing potential applied to the argon atoms. The following conditions and parameters apply:

$K_B = 1.3807\ 10^{-16}\,\text{erg/K}, T_0 = 1200\ \text{K},\ n_e = 7\ 10^{11}\text{cm}^{-3},$

$d = 3.84\ 10^{-8}\text{cm}$

(diameter of the Argon atom), the electron rest mass and charge $m_e = 9.093 \times 10^{-28}\,gm,\ e = 4.8\ 10^{-10}\ esu$ are used to calculate the dimensionless parameter $\alpha_0 = 9.58 \times 10^{-5}$, the electron-electron collision relaxation time $\tau_{ee} = 1.15 \times 10^{-9}$ and the mean free path of the electron gas $\lambda = \frac{1}{\sqrt{2}\pi n_e d^2}$ $= 2.180\ 10^2\,cm$ compared to the electron Debye length $\lambda_{De} = \sqrt{\frac{K_B T_0}{4\pi n_e e^2}} = 2.85\ 10^{-5}\ cm$, $f_1 = 1.66667$. Using the idea of the shooting numerical calculation method, we evaluate the transformation constants to obtain $m = 1$, $l = 0.8$ and the plate Mach number $Ma = 10^{-1}$.

All the variables of the problem satisfy the initial and the boundary conditions, Eq.(21), of the problem, see Figures (2, 3, 5, 6). This cannot be exactly examined in the previous study [6], because of the discontinuity in the.

Figure (1) shows that the deviation from equilibrium is small and in a course of time the perturbed velocity distribution functions F_1 and F_2 approach to equilibrium velocity distribution function F_0 as (y=0.001), i. e. in the region near the moving plate, which represents the region that we are interested for studying. This gives a good agreement with the famous Le Chatelier principle. Figure (2) illustrated that, the mean velocity of electrons, near the moving plate, has a maximum value equal the Mach number (=Ma) of the plate, which satisfies the condition of the problem. The behavior of the shear stress compatibles with the behavior of the velocity itself, see Figure (3).

The viscosity coefficient [30] for a rarefied gas in a slaw fellow is $\mu = \frac{\tau_{xy}}{\partial V_x / \partial y}$. Its behavior is represented in Figure (4), which shed light upon the resistant to the motion, which increases with time. This is because if any change is imposed on a system that is in equilibrium then the system tends to

adjust to a new equilibrium counteracting the change. Near the moving plate, Figures (5, 6) indicate the behavior of the self-generated fields. The induced electric field decreases with time while the induced magnetic field increases with it adjacent to the plate. The induced electric field increases with time while the induced magnetic field decreases with it far from the plate. Clearly, they satisfy the conditions of the problem.

The entropy S increases with time, which gives a good agreement with the second law of thermodynamics, see Figure (7). The entropy production behavior is fulfilled the famous Boltzmann H-Theorem, where $(\sigma \geq 0)$ for all values of y and t, see Figure (8). The thermodynamic force, corresponding to the velocity of the moving plate, has a maximum value equal the Mach number (=Ma) of the plate, which satisfies the condition of the problem., see Figure (9).

Upon passing through a plasma, a charged particle (electron) looses (or gains) part of its energy because of the interaction with the surroundings because of plasma polarization and collisions [35]. The energy loss (or gain) of an electron is determined by the work of the forces acting on the electrons in the plasma by the electromagnetic field generated by the moving particle itself, since the suddenly moving plate causes work to be done on the gas, changing the internal energy of the gas U. As seen in Figures (10–13), the change in the internal energy due to the variation of entropy and paramagnetic is smoothly dampened with time by energy lost to and gained from the ions and plate, respectively. While the change in internal energy increases gradually with time because of the intensive variables, corresponding to either polarization or diamagnetic plasma.
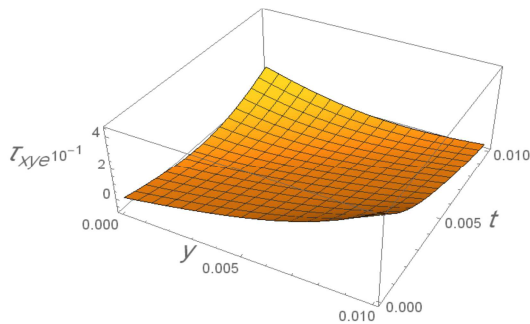
6. Conclusions
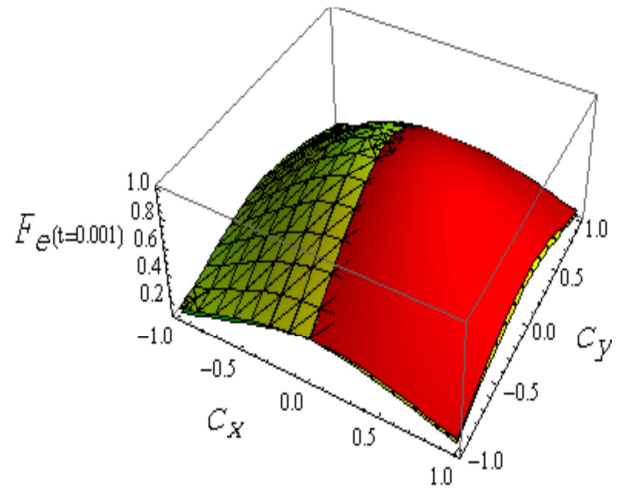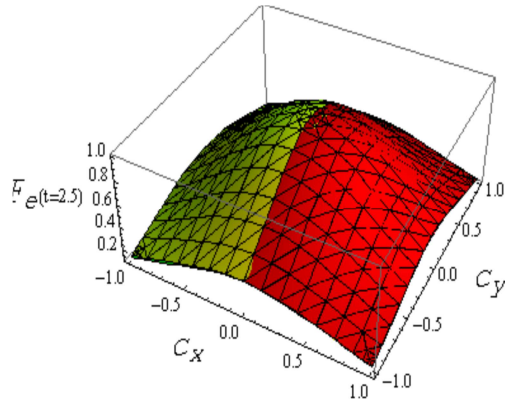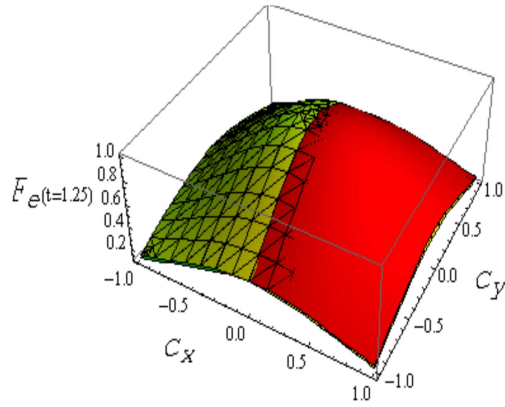
Figure 1. *The comparison between the combined perturbed velocity distribution functions for electrons F_e [F_1 (gray), F_2 (black)] and electrons equilibrium velocity distribution function F_o (grid) at (t = 0.001, 1.25 and 2.5) for a fixed y value (0.001) with the Mach number of the plate Ma = 0.1.*
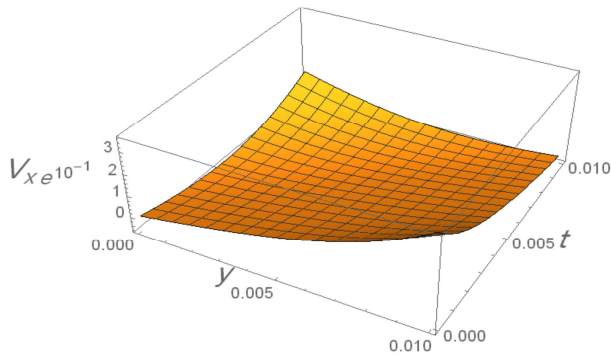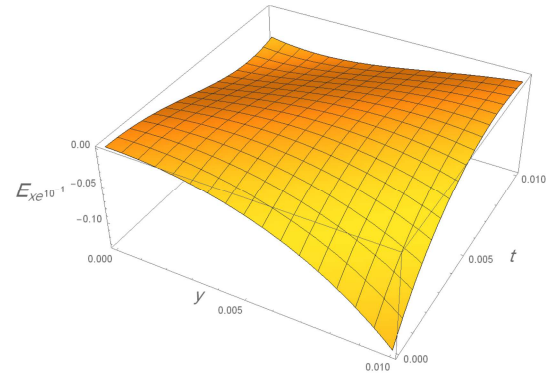

Figure 2. *The velocity V_x versus space y and time t.*
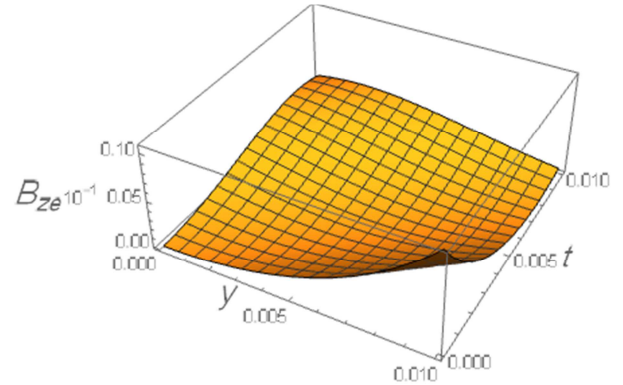

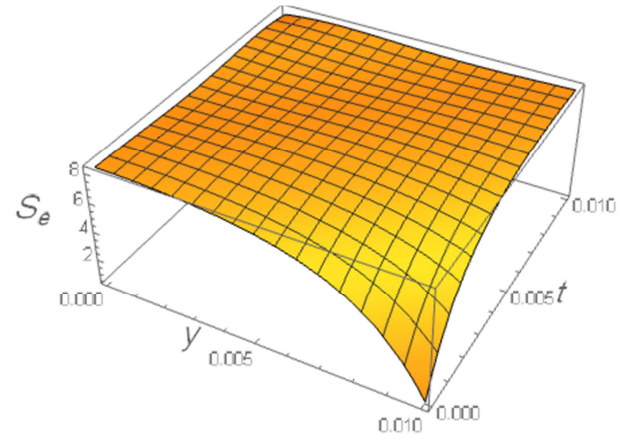**Figure 3.** *The shear stress τ_{xy} versus space y and time t.*
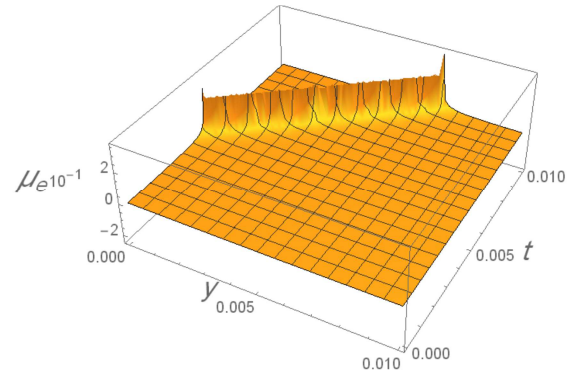
Figure 4. *The viscosity coefficient versus space y and time t.*

Figure 5. *The induced electric field versus space y and time t.*

Figure 6. *The induced magnetic field versus space y and time t.*
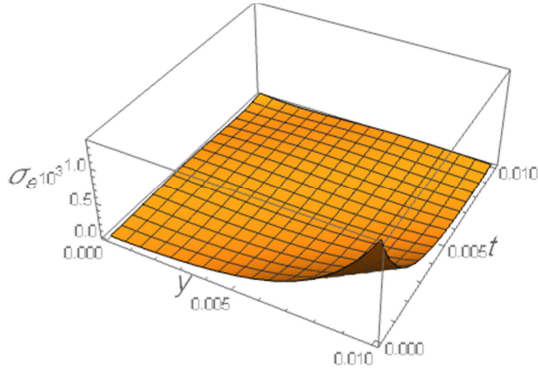

Figure 7. *Entropy S versus space y and time t.*

Figure 8. *Entropy production σ versus space y and time t.*
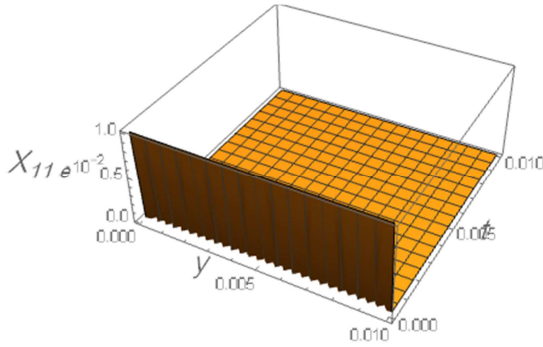

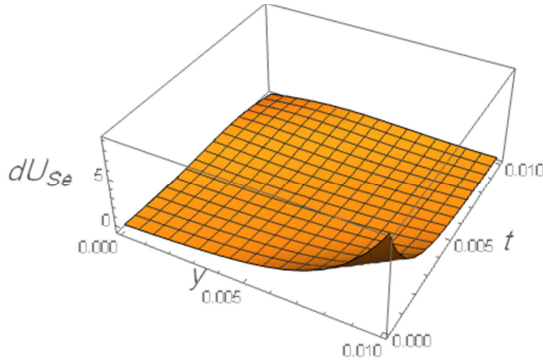**Figure 9.** *thermodynamic force X_{11} versus space y and time t.*

Figure 10. *Internal energy change dU_S versus space y and time t.*

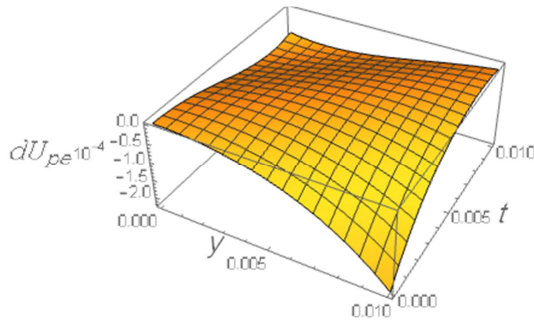**Figure 11.** *Internal energy change dUpol versus space y and time t.*

6. Conclusions

The solution of the unsteady BGK Boltzmann kinetic equation in the case of a rarefied electron gas using the method of the moments of the two-sided distribution function together with Maxwell's equations is developed within the travelling wave exact solution method and the exact value of electron-electron, electron-ion and electron-neutral collision frequencies. This solution allows for the calculation of the components of the velocity of the flow. By inserting them into the suggested two-sided distribution functions and applying the Boltzmann H-theorem, we can evaluate the entropy, entropy production, thermodynamic force, and kinetic coefficient. Via Gibbs' equations, the ratios between the different contributions of the internal energy change are evaluated based upon the total derivatives of the extensive parameters. The predictions, estimated using Gibbs' equations, reveal the following order of maximum magnitude ratios between the different contributions to the internal energy change, based on the total derivatives of the extensive parameters:

$$dU_S \; : \; dU_{pol} \; : \; dU_{dia} = 1 \; : \; 10^{-4} \; : \; 10^{-3}$$

$$dU_S \; : \; dU_{pol} \; : \; dU_{par} = 1 \; : \; 10^{-4} \; : \; 10^{-3}$$

It is concluded that the effect of the changes of the internal energies dU_{pol}, dU_{dia}, and dU_{par} due to electric and magnetic fields are very small in comparison with dU_S, in recognition of the fact that these fields are self-induced by the sudden motion of the plate.
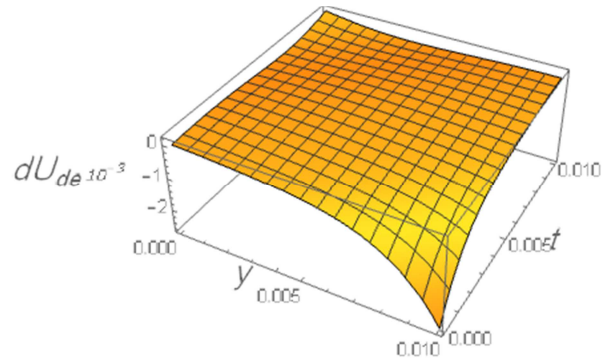
Acknowledgements

Figure 12. *Internal energy change dU_{dia} versus space y and time t.*
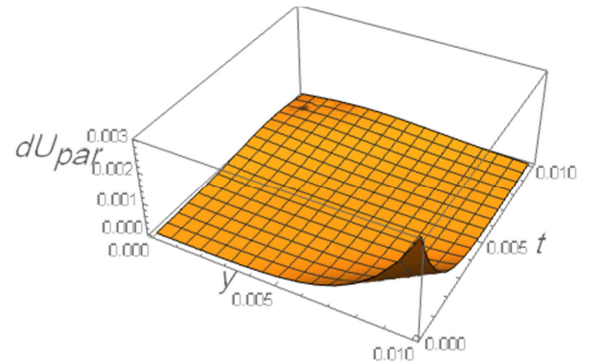
Figure 13. *Internal energy change dUpar versus space y and time t.*

Nomenclature

B	The induced magnetic field vector
B	The induced magnetic field
E	The induced electric vector
E	The induced electric field
F	The velocity distribution function
F_0	The local Maxwellian distribution unction
F_1	Distribution function for going downward particles $c_y < 0$
F_2	Distribution function for going upward particles $c_y > 0$
J	The current density
$J_y^{(S)}$	The entropy flux component along y-axis direction
K_B	Boltzmann constant (Erg/K°) $1.38\ 10^{-16}$
L_{11}	The kinetic coefficient
Ma	The plate Mach number
M	Specific magnetization
P	Polarization
R	The gas constant
S	Entropy per unit mass
T	The temperature
U	The internal energy of the gas
U_0	Plate initial Velocity
V_x	The mean velocity
V_{x1}	The mean velocity related to F_1
V_{x2}	The mean velocity related to F_2
V	Gas volume
V_{Te}	Thermal velocity of electrons
V_{Ti}	Thermal velocity of ions
X_{11}	Thermodynamic force
c_0	The speed of light
c	The velocity of the particles
d	particle diameter
e	The electron charge
f	Lorantz's force vector
m_e	electron mass
m_i	ion mass
n	The mean density
n_e	electrons concentration
n_i	ions concentration
p	Pressure
$\vec{r}$	The position vector of the particle
t	time variable
$\vec{u}$	The mean velocity of the particle
dU_S	The internal energy change due to the variation of entropy
dU_{Pol}	The internal energy change due to the variation of polarization
dU_{par}	The internal energy change due to the variation of magnetization
dU_{dia}	The internal energy change due to the variation of the induced magnetic field
y	displacement variable
Z	Ionization
Superscripts	
'	Dimensionless variable
Subscripts	
e	Related to electrons
i	Related to ions
eq	Equilibrium
Greek letters	

τ	The relaxation time
τ_{xy}	The shear stress
μ	Viscosity coefficient
λ	The mean free path
α_0	Dimensionless parameter
β	Damping constant
ν	Collision frequency
ε	Mass ratio
λ	mean free path
λ_D	Debye radius

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